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ONE-DIMENSIONAL MOTION OF A MAGNETIZABLE CONDUCTING GAS

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A steady one-dimensional adiabatic flow of a magnetizable ideally conducting inviscid perfect gas through a tube in a magnetic field, is considered. A differential equation is obtained determining the conditions under which the gas will be continuously accelerated passing through the speed of sound. The existence of a special divergent-convergent accelerating nozzle is elucidated. It should be noted that sonic and simple waves in a magnetizable compressible medium were studied in [1-3].

1. The adiabatic steady motion of a magnetizable ideally electroconducting inviscid and non-heat-conducting perfect gas, can be described in the ferrohydrodynamic approximation by the following system of equations [3,4]:

$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \rho\nabla\left(\frac{H^2}{8\pi}\frac{\partial\mu}{\partial\rho}\right) - \frac{H^2}{8\pi}\frac{\partial\mu}{\partial T}\nabla T - \left[\frac{\mathbf{B}}{4\pi}\times\operatorname{rot}\mathbf{H}\right]$$

$$\mathbf{v}\nabla\left(s + \frac{H^2}{8\pi\rho}\frac{\partial\mu}{\partial T}\right) = 0, \quad \nabla(\rho\mathbf{v}) = 0 \qquad (1.1)$$

$$\operatorname{rot}[\mathbf{v}\times\mathbf{B}] = 0, \quad \nabla\mathbf{B} = 0$$

where the linear relation $\mathbf{B} = \mu(\rho, T)\mathbf{H}$ connecting the magnetic induction $\mathbf{B}$ and magnetic field strength $\mathbf{H}$ is assumed to hold (standard notation is used). The expression given for the entropy s can be conveniently replaced by the energy equation which can be obtained from the first two equations of (1.1)

$$\mathbf{v}\left\{\left[\frac{\mathbf{B}}{4\pi\rho}\times\operatorname{rot}\mathbf{H}\right] + \nabla\left[\frac{v^2}{2} + w + \frac{H^2}{8\pi\rho}\left(T\frac{\partial\mu}{\partial T} - \rho\frac{\partial\mu}{\partial\rho}\right)\right]\right\} = 0 \qquad (1.2)$$

where w is the gas enthalpy.

We shall limit ourselves to the case of one-dimensional flow in the stream tube, under the assumption that all gas and field parameters depend only on the Cartesian coordinate x directed along the flow. We also assume that the velocity components $v_y = v_z = 0$, $v_x = v$, and the magnetic field components H_x, H_y and H_z coincide with the x, y, z - axes of the Cartesian coordinate system.

Let us write the first, fourth and fifth equation of (1.1) and equation (1.2) for the one-dimensional case in question, and let us supplement them with the equation of state and the condition that the rate of flow through the tube is constant

$$p = \rho RT, \quad \rho vF = \text{const}$$

Here R is the gas constant and F denotes the transverse cross-section of the tube. Solving this system of equations, we obtain the following differential relation:

$$(v^2 - a^2)\frac{dv}{v\,dx} = N\frac{dF}{F\,dx} \tag{1.3}$$

$$a^2 = \frac{a_0^2}{k} + \frac{H_n^2}{2\pi}\left(\frac{\mu}{2\rho} - \frac{\partial\mu}{\partial\rho}\right) + \frac{\rho H^2}{8\pi}\left[\frac{2}{\mu}\left(\frac{\partial\mu}{\partial\rho}\right)^2 - \frac{\partial^2\mu}{\partial\rho^2}\right] +$$

$$(1 - k^{-1})\,a_0^2\left(\Lambda_1 - \frac{H_n^2}{4\pi\rho R}\frac{\partial\mu}{\partial T}\right)^2 \Lambda_2^{-1}, \quad H_n^2 = H_y^2 + H_z^2$$

$$N = \frac{a_0^2}{k} - \frac{H_n^2}{4\pi}\frac{\partial\mu}{\partial\rho} + \frac{\rho H^2}{8\pi}\left[\frac{2}{\mu}\left(\frac{\partial\mu}{\partial\rho}\right)^2 - \frac{\partial^2\mu}{\partial\rho^2}\right] +$$

$$(1 - k^{-1})\,a_0^2\left(\Lambda_1 - \frac{H_n^2}{4\pi\rho R}\frac{\partial\mu}{\partial T}\right)\Lambda_1\Lambda_2^{-1}$$

$$\Lambda_1 = 1 + \frac{H^2}{8\pi R}\left(\frac{2}{\mu}\frac{\partial\mu}{\partial\rho}\frac{\partial\mu}{\partial T} + \frac{\partial\mu}{\rho\,\partial T} - \frac{\partial^2\mu}{\partial\rho\,\partial T}\right),$$

$$\Lambda_2 = 1 + \frac{TH^2}{8\pi\rho c_v}\left[\frac{\partial^2\mu}{\partial T^2} - \frac{2}{\mu}\left(\frac{\partial\mu}{\partial T}\right)^2\right]$$

which represents a novel version of the law of reversal of action [5]. Here a and a_0 denote the speed of sound in the gas, respectively, in the presence and absence of the magnetic field, c_v is the specific heat of the gas at constant volume and k is the adiabatic exponent.

In the limiting case of the standard magneto-gas-dynamics ($\mu = \text{const}$), (1.3) yields an expression [6] describing a motion of a perfectly conducting gas in a channel of variable cross section in a transverse magnetic field.

When $H_n = 0$ ($H_y = H_z = 0$, $H_x = H$), the gas behaves like an electric insulator and (1.3) yields a simpler expression describing the part played by the channel in the case of a nonconducting magnetizable gas.

2. To explain the characteristic features of the flow of a magnetizable conducting gas in a tube (channel), it is expedient to consider particular cases of the relation $\mu = \mu(\rho, T)$. Thus in the case $\mu = \mu(\rho)$ we have

$$a^2 = a_0^2 + \frac{\rho H^2}{8\pi}\left[\frac{2}{\mu}\left(\frac{d\mu}{d\rho}\right)^2 - \frac{d^2\mu}{d\rho^2}\right] + \frac{H_n^2}{4\pi}\left(\frac{\mu}{\rho} - 2\frac{d\mu}{d\rho}\right), \tag{2.1}$$

$$N = a^2 + \frac{H_n^2}{4\pi}\left(\frac{d\mu}{d\rho} - \frac{\mu}{\rho}\right)$$

The condition $a^2 > 0$ is necessary for the flow to be evolutionary, and the sign of the parameter N is immaterial. When $N < 0$, i.e. when

$$\frac{\rho H^2}{8\pi}\left[\frac{d^2\mu}{d\rho^2} - \frac{2}{\mu}\left(\frac{d\mu}{d\rho}\right)^2\right] + \frac{H_n^2}{4\pi}\frac{d\mu}{d\rho} > a_0^2 \tag{2.2}$$

it follows from the first equation of (1.3) that the change in the form of the channel affects the flow in the manner opposite to that caused by the usual nozzle. In order to accelerate the flow continuously through the speed of sound a, the channel must at first be diverged, and then converged after the value a has been reached, i.e. when $N < 0$, the supersonic nozzle must be barrel-shaped. It is evident that when the condition $a^2 > 0$ holds, the case of $N < 0$ is possible only when $(d\mu/d\rho) - \mu\rho^{-1} < 0$ or $\mu > \alpha\rho$ where α is an arbitrary constant.

The above argument can be confirmed by considering the case of magnetic gases with the equation of state $\mu - 1 = \alpha\rho$ [7] for which $\mu > \alpha\rho$. We assume for simplicity that $H_x = 0$. Then the inequality (2.2) will become $\alpha H_n^2 (4\pi\mu)^{-1} > a_0^2$ so that for a paramagnetic gas ($\mu > 1, \alpha > 0$) we can attain, by a suitable choice of the magnetic field strength $H_n^2 > 4\pi\mu\alpha^{-1}a_0^2$, the value $N < 0$ i.e. a specific property of the channel. For a diamagnetic gas ($\mu < 1$, $\alpha < 0$) we always have $N > 0$. The condition of evolutionarity $a^2 > 0$ holds in both cases.

It is interesting to note that in the absence of conductivity (when, as we said before, we must set $H_n = 0$), the first relation of (1.3) and (2.1) together yield

$$\left(\frac{v^2}{a^2} - 1\right)\frac{dv}{v\,dx} = \frac{dF}{F\,dx}, \quad a^2 = a_0^2 + \frac{\rho H_x}{8\pi}\left[\frac{2}{\mu}\left(\frac{d\mu}{d\rho}\right)^2 - \frac{d^2\mu}{d\rho^2}\right] \qquad (2.3)$$

where the expression for a was obtained in [3] by another method. From this it follows that a nonconducting magnetizable gas with $\mu = \mu(\rho)$ is accelerated in the channel like a normal gas, but the "flow crisis" emerges when the stream velocity attains the speed of sound a (2.3). The quantity a may be larger or smaller than a_0 depending on the form of the function $\mu = \mu(\rho)$.

The other particular case $\mu = \mu(T)$ for a conducting magnetizable gas can be investigated in the same way. It is therefore preferable to consider a general case $\mu = \mu(\rho, T)$ in the form of the Curie Law [7] $\mu - 1 = \alpha\rho T^{-1}$. We shall again set $H_x = 0$ for simplicity. Then the first relation of (1.3) in which

$$a^2 = \frac{H_n^2}{4\pi\rho\mu} + \frac{a_0^2}{k}[1 + (k-1)\Lambda^2{}_+\Lambda]$$

$$N = \frac{a_0^2}{k}\left[\Lambda_- + (k-1)\Lambda_+\Lambda^{-1}\left(1 - \frac{\rho\alpha^2 H_n^2}{4\pi\mu R T^3}\right)\right]$$

$$\Lambda_+ = 1 + \frac{\alpha H_n^2}{4\pi\mu R T^2}, \quad \Lambda_- = 1 - \frac{\alpha H_n^2}{4\pi\mu R T^2}, \quad \Lambda = 1 + \frac{\alpha H_n^2}{4\pi\mu c_v T^2}$$

will hold again.

It is clear that for a paramagnetic gas ($\alpha > 0$) the restriction $a^2 > 0$ holds. If in addition $H_n^2 > 4\pi\mu\rho^{-1}\alpha^{-2}RT^3$, the channel will have the specific property noted above. For a diamagnetic gas the nonevolutionarity $a^2 < 0$ may appear.

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